RESISTANCE OF FIBROUS FILTERS IN A SLIP FLOW

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The results of experiments on measurement of the resistance of glass-fiber filters and model filters made of parallel cylinders to the flow of rarefied air are described. The experimental data are compared with Kuwabara's theory [3]. This comparison permits calculating the momentum exchange coefficients of gas molecules with the surface.

1. It was shown in [1, 2] that in the region of self-preserving slip flow the resistance of fibrous filters Δp as a function of the pressure of a gas flow p is described by the equation

$$\frac{\Delta p}{u} = A\left(1 + \frac{B}{p}\right)^{-1} \tag{1.1}$$

where A and B are constants for a given filter and given gas, u is the velocity of the gas flow in front of the filter.

The functional dependence of the parameter A on the physical characteristics of the filter and gas flow was studied in [3-5].

It was found that

$$A = 4\mu h\alpha \,/\, \langle a^2\rangle\, k(\alpha\gamma), \qquad k(\alpha\gamma) = -0.5 \ln\ \alpha\gamma + \alpha\gamma - 0.25\alpha^2\gamma^2 - 0.75 \tag{1.2}$$

Here h is the thickness of the filter, a is the radius of the fiber, α is the portion of the filter volume occupied by fibers, μ is the viscosity, and γ is a structure coefficient.

In conformity with [3, 5]

$$\gamma = 1 \tag{1.3}$$

for a homogeneous system of parallel cylinders and

$$\gamma = 2\,\langle a\rangle^2 \,/\, \pi\,\langle a^2\rangle \tag{1.4}$$

for a polydispersed fan model* (the angular brackets denote averaging).

It is considered [5] that the fan model is a good approximation to real homogeneous filters, the fibers in which are arranged randomly and homogeneously (equivalently) in parallel planes normal to the vector of the mean flow velocity.

Following [3, 6, 7], we can write the theoretical expression for resistance of a monodispersed filter whose fibers experience a Kuwabara slip flow in the form

$$\frac{\Delta p}{u} = A\left\{\frac{1 + 2\xi a^{-1}}{1 + 2\xi a^{-1}\left[1 + \varphi(\alpha\gamma)k^{-1}(\alpha\gamma)\right]}\right\} \tag{1.5}$$

where $\varphi(\alpha\gamma) = -\alpha\gamma + 0.5\alpha^2\gamma^2 + 0.5$, ξ is the slip coefficient, and A is defined in (1.2).

*A. A. Kirsh, Investigations in the Field of Fibrous Aerosol Filters, Dissertation for Candidate of Chemical Sciences Degree, Moscow, 1968.

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TABLE 1. Experimental Data on the Resistance of Model and Glass-Fiber Filters

α	N	h	$\langle a\rangle\cdot 10^4$	B_0	$A_0\cdot 10^4$	$A\cdot 10^4$	σ
0.0057	12		85	0.627	0.549	0.562	0.7
0.0178	11		85	0.885	1.54	1.47	0.697
0.048	12		85	1.25	5.0	5.07	0.705
0.376	11		175	1.82	90.8	47.6	
0.00762		0.338	1.57	22.5	241	244	0.77
0.0171		0.154	1.57	28.8	284	310	0.79

For $\xi a^{-1}\to 0$ Eq. (1.5) is simplified and as applied to polydispersed filters has the form (see Appendix 1)

$$\frac{\Delta p}{u}=\frac{4\mu h\alpha}{\langle a^2\rangle k(\alpha\gamma)}\left[1+2\xi\frac{\langle a\rangle\varphi(\alpha\gamma)}{\langle a^2\rangle k(\alpha\gamma)}\right]^{-1} \tag{1.6}$$

Equation (1.6) is outwardly similar to empirical expression (1.1) and reveals the functional dependence of B on the physical characteristics of the filter and gas flow.

The slip coefficient of gas ξ near a flat wall was estimated in a number of works [8, 9]. For a gas model of solid elastic spheres it was found that

$$\xi=(2\sigma^{-1}-1)\ 1.09l$$
$$l=\mu\,/\,0.499\,\rho c,\quad c=(2kT\,/\,\pi m)^{1/2} \tag{1.7}$$

Here ρ is the gas density, k is the Boltzmann coefficient, T is temperature °K, m is the mass of the gas molecule, and σ is the momentum-exchange coefficient [10].

For a real gas, according to [11], the numerical coefficient in (1.7) should be within 1.09-1.18, where the value 1.18 corresponds to a gas of Maxwellian molecules [9]. If we assume that (1.7) holds true for a real gas near a cylindrical surface, then constant B in (1.1) will have the form

$$B=\left(\frac{2}{\sigma}-1\right)2.18\,l_0p_0\frac{\langle a\rangle}{\langle a^2\rangle}\frac{\varphi(\alpha\gamma)}{k(\alpha\gamma)} \tag{1.8}$$

where p_0 is the pressure for which l_0 is calculated by (1.7).

We need note that consideration of fluctuations of the porosity of a homogeneous filter does not change Eq. (1.6) (see Appendix 2).

The resistance of thin filters inhomogeneous with respect to α is determined by an expression of type (1.6) in which we use in place of the hydrodynamic parameters $k<\alpha>\gamma)$ and $\varphi(<\alpha>\gamma)$, respectively, the parameters

$$k'(\langle\alpha\rangle\gamma,c_1,c_2)=-0.5c_1\ln(\langle\alpha\rangle\gamma)+\langle\alpha\rangle\gamma-0.25\langle\alpha\rangle^2\gamma^2-0{,}75c_1+c_2$$
$$\varphi'(\langle\alpha\rangle\gamma,c_1)=-\langle\alpha\rangle\gamma+0.5\langle\alpha\rangle^2\gamma^2+0.5c_1 \tag{1.9}$$

The coefficients c_1 and c_2 are practically constant quantities for filters with the same character of inhomogeneities. To calculate these coefficients we must know either the probability-density function of α for an arbitrary volume V of the filter or the experimental values of the filter's resistance for two different $<\alpha>$ (the parameter $<\alpha>$ changes when porous material is squeezed). Good specimens of glass-fiber filters are characterized by coefficients

$$1\leqslant c_1\leqslant 1.2,\quad 0\leqslant c_2\leqslant 0.14$$

2. Experiment and Results

1°. We measured the pressure drop during flow of air of varying density through polydispersed fibrous filters and through model filters consisting of parallel cylinders of the same radius. The fibers in the model and real filters were oriented perpendicular to the mean flow velocity vector.

The radii of the fibers in the glass-fiber filters had an approximately lognormal distribution with parameters

$$\langle(\lg a-\lg a')^2\rangle=0.234^2,\quad \lg a'=-3.85$$

The quantities $<\alpha>=1.57\cdot 10^{-4}$ and $<\alpha^2>=2.87\cdot 10^{-8}$ were determined by direct calculation of the results of 160 measurements of the thicknesses of individual fibers.

The model filters were made up of 11-12 round frames of thickness h_1 on which wires coated with insulating lacquer were stretched parallel with spacing h_2. In putting the frames together we were not concerned with obtaining a staggered or corridor spatial structure; it was only necessary that all wires in the model were parallel to each other. For all models the ratio $h_1/h_2 = 1$.

The glass-fiber and model filters had a frontal surface of 3 and 28 cm^2, respectively.

The pressure drop on the filters was measured by a micromanometer with a sensitivity of $2 \cdot 10^{-5}$ torr.

2°. In the pressure region where (1.1) holds true, the experimental data in coordinates $u/\Delta p$, $1/p$ for each filter lay on a straight line, and from the graphs obtained we then calculated the values of A_0 and B_0. The measurement results are presented in Table 1. The theoretical value of A was calculated by Eq. (1.2); in calculating A for the models the quantity $h\alpha/\pi a^2$ in (1.2) was replaced by N/h_2, where N is the number of frames in the model.

The coefficient σ was calculated by Eq. (1.8) for each experimental value of B_0.

In Table 1 the linear parameters are expressed in centimeters, and pressure in millimeters of mercury.

3°. In the experiments with glass-fiber filters the experimental points did not deviate from the average linear dependence $u/\Delta p$, $1/p$ by more than 2% in the interval $0 \le l/\langle a \rangle \le 3$ and by more than 4% in the interval $0 \le l/\langle a \rangle \le 12$ (2% is the experimental error).

For the model filters the corresponding values of max $l/\langle a \rangle$ are one-third as much, possibly owing to the smaller accuracy of the experiments at small p. Thus, for the investigated filters the dependence (1.6) obtained under the condition $\xi \to 0$ describes formally the experimental results for finite values of ξ, penetrating into the transitional region of flows. For a single cylinder the region of existence of slip flow is estimated [10] as $0 \le l/a \le 0.2$.

4°. The constancy of the calculated values of σ for the model and real filters in the region $0.0057 \le \alpha \le 0.05$ indicates a regular functional dependence of B on α in (1.8). The concrete values of σ, equal to 0.7 for lacquer and 0.78 for glass, differ from the corresponding values 0.79 and 0.89 given in [10]. This is related, first, with the fact that Kuwabara's model probably does not give a suitable linear numerical coefficient of ξ in (1.6) and, second, with the use of different numerical coefficients in determining ξ itself.

For the experiments we selected the most homogeneous glass-fiber filters having the best agreement between the theoretical and experimental values of A. We note that according to (1.9) the parameter B is less sensitive to inhomogeneity of the filters than A, and therefore in estimating σ a correction for inhomogeneity was not made.

Appendix 1. The experimental result (1.4) signifies that in the model polydispersed fan filter, fibers with different diameters experience, on the average, the same force from the flow per unit length. In other words, in Kuwabara's problem [3], solved for a polydispersed system of cylinders, the ratio a/b should remain constant for any a, where b is the radius of a concentric fiber of an imaginary cylinder on whose surface Kuwabara placed external boundary conditions. According to (1.4), for $\xi = 0$

$$b = a\left(\pi \langle a\rangle^2 / 2\alpha \langle a^2\rangle\right)^{1/2}$$

We extend this result to slip flow (provided that a part of the thinnest fibers in the filter does not enter the region of intermediate or molecular flow). For $\alpha \ll 1$ the condition of equality of forces on fibers of different diameters is written in the form

$$k\left(\frac{a^2}{b^2}\right)\left[1 + 2\xi\varphi\left(\frac{a^2}{b^2}\right) \Big/ ak\left(\frac{a^2}{b^2}\right)\right] \approx -0.5 \ln\left(\frac{a^2}{b^2}\right) - 0.75 + \frac{\xi}{a} = \text{const}$$

and the condition of constancy of the filter's geometry is written accordingly in the form

$$\langle b^2(a, \xi)\rangle = \pi \langle a^2\rangle^2 / 2\alpha \langle a\rangle^2$$

Expanding the first expression relative to b, then using the second expression for determining const, and, finally, summing the force obtained on the entire length of the fibers per unit surface of the filter, we obtain (1.6).

Appendix 2. Let be given a thin filter with an infinite surface and parameters h, $\langle\alpha\rangle$, $\langle a\rangle$, $\langle a^2\rangle$. We isolate in it volume V in the form of a washer of radius r and height h whose bases are parallel to the surface of the filtering material. If the filter consists of infinite rectilinear fibers, the fluctuations of density α in this washer under the conditions $(m_i - \langle m\rangle)^3/\langle m\rangle^2 \to 0$ and $m \to \infty$ are determined by the expression

$$\mathrm{Prob}\{\alpha_1 \leqslant \alpha < \alpha_2 \mid V, \langle\alpha\rangle\} = 0\ 5\,[\Phi(y_{m_2 \pm 0.5}) - \Phi(y_{m_1 - 0.5})]$$

$$\Phi(y) = \left(\frac{2}{\pi}\right)^{1/2} \int_0^y \exp\left(\frac{-x^2}{2}\right) dx, \quad y_{m_i \pm 0.5} = \frac{m_i \pm 0.5 - \langle m\rangle}{\sqrt{\varkappa \langle m\rangle}}$$

$$m_i = 2\alpha_i V / \pi^2 r \langle a^2\rangle$$

Here $\langle m\rangle$ is the average number of fibers intersecting V, the coefficient $\varkappa = 1$ is for homogeneous filters and $\varkappa > 1$ is for filters with a small deviation from homogeneity.

For a given pressure drop Δp on the filter the local mean velocity of the gas flow u' through the washer is established with consideration of the effect of the velocity u of the adjacent gas flow through the section of the filter adjacent to the washer (this effect is considerable only for a packing density of the washer $\alpha \to 0$). To determine u' we can write the following expressions:

$$\Delta p = 8\mu h r^{-2}(u' - u) + 4\mu h \alpha u' / \langle \alpha^2\rangle k(\alpha\gamma)$$
$$u = \Delta p \langle a^2\rangle k(\langle\alpha\rangle\gamma) / 4\mu h \langle\alpha\rangle, \quad r \geqslant l'$$
$$l' = \pi^2 \langle a^2\rangle (1 - \langle\alpha\rangle) / 4 \langle a\rangle \langle\alpha\rangle$$

Here l' denotes the mean free path of a ray in the pores of the filter in a plane parallel to the fibers. Calculation shows that the flow velocity through the washer, averaged with respect to all fluctuations of α, exceeds the value of u only by thousandths of a percent if we consider the coefficient $\varkappa = 1$.

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